

Exam II , MTH 101, Spring 2015

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QUESTION 1. (i) **(6 points)** A kindergarten school transfers its 130 kids home every day. It uses two types of busses: mini-buses and regular buses. A mini-bus can carry 10 kids, where a regular bus can carry 40 kids. The school will pay 100 Dhs daily for each mini-bus and it will pay 310 Dhs daily for each regular bus. How many buses of each type should the school rent in order to minimize its daily expenses?

x = # of mini-bus
y = # of regular bus

$C = 100x + 310y$
Restriction: $10x + 40y = 130$

$10x + 40y = 130$
$10x = 130 - 40y$
$x = 13 - 4y$

y	x	Cost
0	13	1,300
1	9	1,210
2	5	1,120
3	1	1,030

Minimum cost is $1,030 & it occurs when 1 mini-bus & 3 regular buses are used.

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(ii) **(6 points)** State only the dual problem for the following minimizing question (Do not attempt to solve it) Minimize $C = 20x_1 + 10x_2 + 5x_3$ subject to

$$2x_1 + x_2 + x_3 \geq 10$$
$$x_2 + 4x_3 \geq 22$$

$(x_1, x_2, x_3 \geq o)$

$$A = \begin{array}{c} \\ y_1 \\ y_2 \\ C \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & P \end{array} \\ \left[\begin{array}{ccc|c} 2 & 1 & 1 & 10 \\ 0 & 1 & 4 & 22 \\ \hline 20 & 10 & 5 & 1 \end{array}\right] \end{array}$$

$$A^T = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ P \end{array} \begin{array}{c} \begin{array}{ccc} y_1 & y_2 & C \end{array} \\ \left[\begin{array}{cc|c} 2 & 0 & 20 \\ 1 & 1 & 10 \\ 1 & 4 & 5 \\ \hline 10 & 22 & 1 \end{array}\right] \end{array}$$

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$P = 10y_1 + 22y_2$

subject to Restrictions:

$$\begin{cases} 2y_1 \leq 20 \\ y_1 + y_2 \leq 10 \\ y_1 + 4y_2 \leq 5 \end{cases}$$

$(y_1, y_2 \geq 0)$

$\rightarrow 2y_1 + x_1 = 20$
$y_1 + y_2 + x_2 = 10$
$y_1 + 4y_2 + x_3 = 5$
$P - 10y_1 - 22y_2 = 0$

(iii) **(12 points)** An ice-cream shop makes two types of ice-cream: Chocolate ice-cream and vanilla ice-cream. Number of chocolate ice-cream plus twice number of vanilla ice-cream (in kilograms) must be at least 20 kilograms, number of chocolate ice-cream must be at most twice as many as number of vanilla ice-cream, and number of vanilla ice-cream is at most 8 kilograms. If the shop makes a 120 Dhs profit on each kilogram of Chocolate ice-cream and a 100 Dhs profit on each kilogram of vanilla ice-cream, how many kilograms of each type must the shop make in order to maximize its profit?

Ice cream

vanilla → $x = \#$ of vanilla

chocolate → $y = \#$ of chocolate

$P = 120y + 100x$

restrictions
$$\begin{cases} y + 2x \geq 20 \\ y \leq 2x \\ x \leq 8 \end{cases}$$
$(x, y \geq 0)$

* $y + 2x \geq 20$

→ $y + 2x = 20$

x-int $(y=0)$: $2x = 20$, $x = 10$, $(10, 0)$

y-int $(x=0)$: $y + 2(0) = 20$, $y = 20$, $(0, 20)$

* $x \leq 8$

→ $x = 8$

Excellent

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* $y \leq 2x$

$y - 2x \leq 0$

→ $y - 2x = 0$

x-int $(y=0)$: $-2x = 0$, $x = 0$, $(0, 0)$

y-int $(x=0)$: $y - 2(0) = 0$, $y = 0$, $(0, 0)$

use $y = 2$ & solve for x:

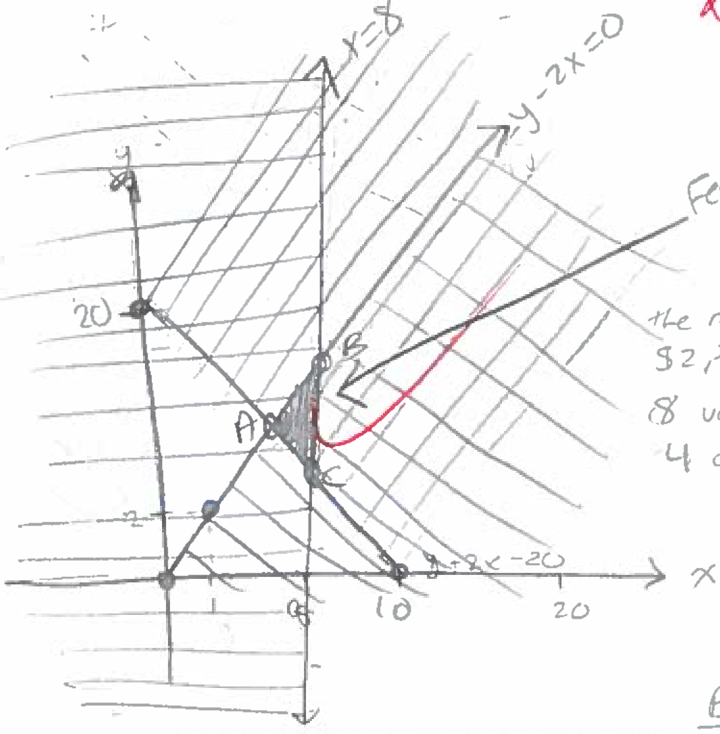
$-2x + 2 = 0$

$-2x = -2$

$x = \frac{-2}{-2}$

$x = 1$

$(1, 2)$



Feasible Region (Bounded)

In order to get the maximum profit of \$2,720, we should sell 8 vanilla-icecreams & 4 chocolate-icecreams

	Points	$P = 100x + 120y$
A	(5,10)	\$1,700
B	(8,16)	\$2,720
C	(8,4)	\$1,280

B: $y - 2x = 0$, $x = 8$

→ $y - 2(8) = 0$

$y = 16$

C: $y + 2x = 20$, $x = 8$

→ $y + 2(8) = 20$

$y = 20 - 16$

$y = 4$

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use (0,0):

$y + 2x \geq 20$

$(0) + 2(0) \geq 20$

FALSE

$x \leq 8$

$0 \leq 8$

True

use (2,1):

$y \leq 2x$

$1 \leq 2(2)$

$1 \leq 4$ TRUE

→ $y - 2x \leq 0$

$1 - 2(2) \leq 0$

$1 - 4 \leq 0$

$-3 \leq 0$

A: $y + 2x = 20$, $y - 2x = 0$

$C = \begin{bmatrix} 2 & 1 \\ -2 & 1 \end{bmatrix}$

$\det(C) = (2)(1) - (-2)(1) = 4$

$x = \frac{\det\begin{bmatrix} 20 & 1 \\ 0 & 1 \end{bmatrix}}{\det(C)} = \frac{(20)(1) - (1)(0)}{4} = 5$

$y = \frac{\det\begin{bmatrix} 2 & 20 \\ -2 & 0 \end{bmatrix}}{\det(C)} = \frac{(2)(0) - (-2)(20)}{4} = 10$

(iv) **(12 points)** Maximize $P = 60x_1 + 20x_2 + 40x_3$ subject to

$$x_1 + x_2 + 0.5x_3 \le 30$$

$$2x_2 + x_3 \le 20$$

$$2x_1 + 3x_2 + x_3 \le 80$$

$(x_1, x_2, x_3 \ge 0)$

$x_1 + x_2 + 0.5x_3 + S_1 = 30$

$2x_2 + x_3 + S_2 = 20$

$2x_1 + 3x_2 + x_3 + S_3 = 80$

$P - 60x_1 - 20x_2 - 40x_3 = 0$

pivot column

b.v.	x_1	x_2	x_3	S_1	S_2	S_3	C
S_1	1	1	0.5	1	0	0	30
S_2	0	2	1	0	1	0	20
S_3	2	3	1	0	0	1	80
P	-60	-20	-40	0	0	0	0

30/1 = 30
80/2 = 40

S_1 exit
x_1 enter

$-2R_1 + R_3 \to R_3$
$60R_1 + R_4 \to R_4$

pivot column

b.v.	x_1	x_2	x_3	S_1	S_2	S_3	C
x_1	1	1	0.5	1	0	0	30
S_2	0	2	1	0	1	0	20
S_3	0	1	0	-2	0	1	20
P	0	40	-10	60	0	0	1,800

30/0.5 = 60
20/1 = 20

S_2 exit
x_3 enter

$-0.5R_2 + R_1 \to R_1$
$10R_2 + R_4 \to R_4$

b.v.	x_1	x_2	x_3	S_1	S_2	S_3	C
x_1	1	0	0	1	-0.5	0	20
x_3	0	2	1	0	1	0	20
S_3	0	1	0	-2	0	1	20
P	0	60	0	60	10	0	2,000

Maximum Profit is \$2,000 & it occurs when:

$x_1 = 20$
$x_2 = 0$
$x_3 = 20$

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(v) **(12 points)** You want to bring exactly 22 small Pizza for your birthday-party. Three types of Pizza are available: Cheese Pizza and each cost 20 Dhs, vegetarian Pizza and each cost 22 Dhs, and Beef Pizza and each cost 21 Dhs. Number of Cheese Pizza must be three times as many as number of the Beef Pizza. You have exactly 456 Dhs to spend on Pizza, how many Pizza of each type can you buy?

let x_1 be cheese pizza

let x_2 be vegetarian pizza

let x_3 be beef pizza

sum of pizza = 22

cheese = \$20

vegetarian = \$22

beef = \$21

total cost = \$456

cheese pizza is 3 times the beef pizza

$x_1 = 3x_3$

$$\boxed{\begin{aligned} x_1 + x_2 + x_3 &= 22 \\ 20x_1 + 22x_2 + 21x_3 &= 456 \\ x_1 - 3x_3 &= 0 \end{aligned}}$$

$$\begin{array}{cccc} x_1 & x_2 & x_3 & C \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 22 \\ 20 & 22 & 21 & 456 \\ 1 & 0 & -3 & 0 \end{array}\right] \quad \begin{array}{l} -20R_1 + R_2 \to R_2 \\ -R_1 + R_3 \to R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 22 \\ 0 & 2 & 1 & 16 \\ 0 & -1 & -4 & -22 \end{array}\right] \quad \tfrac{1}{2}R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 22 \\ 0 & 1 & \frac{1}{2} & 8 \\ 0 & -1 & -4 & -22 \end{array}\right] \quad \begin{array}{l} -R_2 + R_1 \to R_1 \\ R_2 + R_3 \to R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & 14 \\ 0 & 1 & \frac{1}{2} & 8 \\ 0 & 0 & -7/2 & -14 \end{array}\right] \quad -\tfrac{2}{7}R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & 14 \\ 0 & 1 & \frac{1}{2} & 8 \\ 0 & 0 & 1 & 4 \end{array}\right] \quad \begin{array}{l} -\frac{1}{2}R_3 + R_1 \to R_1 \\ -\frac{1}{2}R_3 + R_2 \to R_2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 12 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

∴ I must buy 12 cheese pizzas, 6 vegetarian pizzas and 4 beef pizzas.

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Faculty information

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(v) **(12 points)** You want to bring exactly 22 small Pizza for your birthday-party. Three types of Pizza are available: Cheese Pizza and each cost 20 Dhs, vegetarian Pizza and each cost 22 Dhs, and Beef Pizza and each cost 21 Dhs. Number of Cheese Pizza must be three times as many as number of the Beef Pizza. You have exactly 456 Dhs to spend on Pizza, how many Pizza of each type can you buy?

Pizzas = 22

Cheese $20 ↳ x

Vegeterian $22 ↳ y

Beef $21 ↳ z

$$\begin{cases} 20x + 22y + 21z = 456 \quad (\text{cost}) \\ x + y + z = 22 \\ x = 3z \end{cases}$$

$\rightarrow 20(3z) + 22y + 21z = 456$

$81z + 22y = 456$

$\rightarrow 3z + y + z = 22$

$4z + y = 22$

$y = 22 - 4z$

interesting approach!!

OK but why not using matrices

z	y	c = 456
0	22	484
1	18	477
2	14	470
3	10	463
4	6	456
5	2	449

$x + 6 + 4 = 22$

$x = 22 - 10$

$x = 12$

$\longrightarrow x = 12, \; y = 6, \; z = 4$

check: * $20(12) + 22(6) + 21(4)$

$= 240 + 132 + 84$

$= 456$ ✓

* $(12) + (6) + (4)$

$= 22$ ✓

* $x = 3(4)$

$= 12$ ✓

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Since I did not ask you to use Jordan-Gauss-elimination

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